CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF SCIENCE

PHYS 232: WAVES AND OSCILLATIONS **TIME: 2 HRS**

Streams: BED (SCI) and BSC **DAY/DATE:………………………**

INSTRUCTIONS:

- Answer Question One and any other Two Questions.
- Do not write anything on the question paper
- This is a closed book exam, No reference materials are allowed in the examination room
- There will be No use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely
- You may use the data below.
    i. Gravitational Constant: $g = 10\ ms^{-2}$
    ii. $\sin(A+B) + \sin A - B) = 2\sin A\cos B$

QUESTION ONE (30 Marks)

a. Define the following terms;
    i. Simple harmonic motion **(3 marks)**
    ii. Displacement and Amplitude **(2 marks)**
    iii. Normal modes of oscillation **(1 mark)**
    iv. Logarithmic decrement **(1 mark)**
    v. Resonance **(1 mark)**
    vi. Degrees of freedom **(1 mark)**

b. Write down the linear differential equation of motion for a free simple harmonic oscillator:
    i. In terms of m, the oscillator mass and k, the oscillator stiffness, **(1 mark)**
    ii. In terms of ω_n. **(1 mark)**

c. List the three mechanisms responsible for energy loss of a harmonic oscillator. Which mechanism is used in our analysis of free damped simple harmonic motion and why? **(4 marks)**

d. You are given the linear second order differential equation with constant coefficients m, k, and c, $m\ddot{x} + c\dot{x} + kx = 0$.

i. If the trial solution is $x = \exp(rt)$ then write down the characteristic equation of the above differential equation. **(1 mark)**

ii. Solve the above characteristic equation in r that you have written, and obtain an expression for the roots of this characteristic equation in terms of m, k, and c. **(1 mark)**

iii. Let us define two quantities $\xi = \frac{c}{2m\omega_n}$ and $\omega_n^2 = \frac{k}{m}$ then rewrite the expression for the roots, r, in terms of ω_n and ξ. **(2 marks)**

e. The displacement of a free harmonic oscillator can be expressed as $x(t) = Asin(\omega t + \varphi)$; Use the displacement function to show that the acceleration amplitude = $A\omega^2$ **(2 marks)**

f. What is a wave and how can we classify waves generally according how they are propagated by media? **(3 marks)**

g. A steel wire of 0.01 kg mass and 2 m length is stretched to a tension of 10 N.

i. Calculate the linear frequency of the fundamental vibrations. **(3 marks)**

ii. If the displacement amplitude of the fundamental is 0.02 m at the center of the wire, what is the total energy of the fundamental mode of vibration? **(1 mark)**

iii. What is the velocity amplitude at a point 0.5 m from either end of the wire? **(2 marks)**

QUESTION TWO (20 MARKS)

A block of mass m is connected to two springs of force constants k_1 and k_2 as shown in Figure 1 (a and b), the block moves on a frictionless table after it is displaced from equilibrium and released.

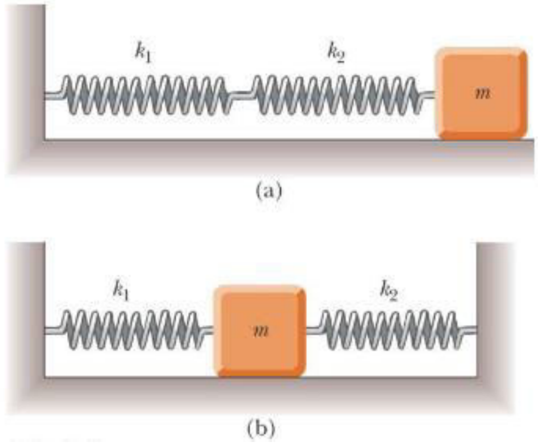

Figure 1

a. Show that in case a the block exhibits simple harmonic motion with period $T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

(4 marks)

b. Show that in case b the block exhibits simple harmonic motion with period $T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$

(4 marks)

c. Consider the following system show in figure 2.

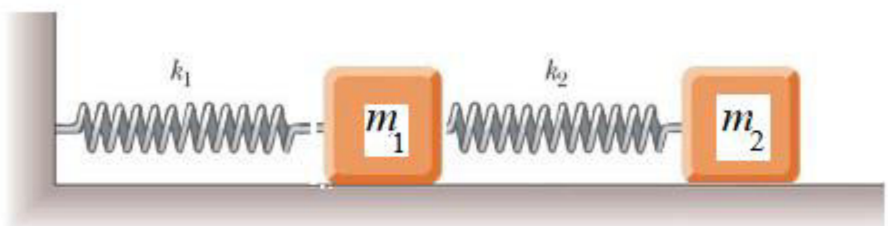

Figure 2

i. By setting the equation of motion for both masses m_1 and m_2 derive the frequency equation $\{m_1m_2\omega^4 - \omega^2[m_2(k_1+k_2) + m_1k_2] + [(k_1+k_2)k_2 - k_1 + k_2^2]\} = 0$ **(9 marks)**

ii. Show that the frequency equation has two solutions (modes of vibrations). **(3 marks)**

QUESTION THREE (20 MARKS)

Consider an element dx of string of length L, which is deformed to length ds by the application of a tension T. If the string has a mass per unit length of ρ .

a. Derive expressions for the kinetic energy per wavelength K_λ and the potential energy per wavelength U_λ , and hence write down the expression for the total energy per wavelength E_λ .

hint: you may use the identity $ds = [(dx)^2 + (dy)^2]^{\frac{1}{2}} \approx dx\left[1 + \left(\frac{\partial y}{\partial x}\right)^2\right]^{\frac{1}{2}} \approx 1 + \frac{1}{2}(\frac{\partial y}{\partial x})^2$

(8 marks)

b. If a wave is described by $y(x,t) = A\sin 2\pi\nu\left(t - \frac{x}{c}\right)$ for a string, show that the mean power input $P = \frac{u_0^2 T}{2c}$. Given that $\int_{t=0}^{t=\tau} cos^2\, 2\pi\nu t\, dt = \frac{1}{2}\tau$, $\int_{t=0}^{t=\tau} dt = \tau$ and $P = \frac{\int_{t=0}^{t=\tau} P_{inst} dt}{\int_{t=0}^{t=\tau} dt}$ **(12 marks)**

QUESTION FOUR (20 MARKS)

a. List the three types of motion possible by the damped system and write their general displacement functions. **(3 marks)**

b. What is the difference between free simple harmonic motion and damped simple harmonic motion in terms of amplitude? **(2 marks)**

c. A body of mass 5.5 kg is hung on a spring of stiffness 1000 Nm^{-1}. It is pulled down 50 mm below the position of static equilibrium and released so that it executes vertical vibrations. There is a viscous damping force acting on the body, of 40 N when the velocity is 1 ms^{-1}. Determine the differential equation of motion and obtain the expression for the displacement of the body as a function of time. **(15 marks)**

QUESTION FIVE (20 MARKS)

a. Using the law of conservation of energy, establish the equation of motion of a body of mass m attracted to the origin O with a force $F = -Kx$. **(3 marks)**

i. What is the frequency of oscillations if m = 100g and K=4.0 x 10^3 N/m? **(1 mark)**

ii. What is the maximum displacement if the mass is brought to the position x = 5 cm and thrown with a velocity 10 m/s in the positive x direction? **(5 marks)**

b. A U-tube open at both ends is filled with an incompressible fluid of density ρ. The cross-sectional area A of the tube is uniform and the total length of the fluid in the tube is L. A piston is used to depress the height of the liquid column on one side by a distance x, (raising the other side by the same distance) and then is quickly removed (Figure 3). What is the angular frequency of the ensuing simple harmonic motion? Neglect any resistive forces and at the walls of the U-tube. **(11 marks)**

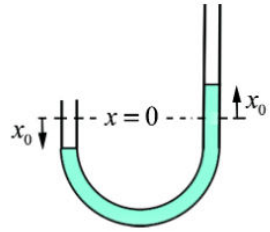

Figure 3